

Table 15.2 ('000 Rs.)

Product Type	Alternative Demand		
	D_1	D_2	D_3
A	14	66	118
B	13	77	141
C	1	73	145

Example 15.2 A food products company is contemplating the introduction of a revolutionary new product with new packaging or replace the existing product at much higher price (S_1) or a moderate change in the composition of the existing product with a new packaging at a small increase in price (S_2) or a small change in the composition of the existing product except the word 'New' with a negligible increase in price (S_3). The three possible states of nature or events are: (i) high increase in sales (N_1), (ii) no change in sales (N_2) and (iii) decrease in sales (N_3). The marketing department of the company worked out the payoffs in terms of yearly net profits for each of the strategies of three events (expected sales). This is represented in the following table:

Strategies	States of Nature		
	N_1	N_2	N_3
S_1	7,00,000	3,00,000	1,50,000
S_2	5,00,000	4,50,000	0
S_3	3,00,000	3,00,000	3,00,000

Which strategy should the concerned executive choose on the basis of

- (i) Maximin criterion
- (ii) Maximax criterion
- (iii) Minimax regret criterion
- (iv) Laplace criterion?

Solution The payoff matrix is rewritten as follows:

(i) **Maximin Criterion:**

States of Nature	S_1	S_2	S_3
N_1	7,00,000	5,00,000	3,00,000
N_2	3,00,000	4,50,000	3,00,000
N_3	1,50,000	0	3,00,000
Column minimum	1,50,000	0	3,00,000 ← Maximin

The maximum of column minima is 3,00,000. Hence, the company should adopt strategy S_3 .

(ii) **Maximax Criterion:**

States of Nature	S_1	S_2	S_3
N_1	7,00,000	5,00,000	3,00,000
N_2	3,00,000	4,50,000	3,00,000
N_3	1,50,000	0	3,00,000
Column maximum	7,00,000	5,00,000	3,00,000
	↑ Maximax		

The maximum of column maxima is 7,00,000. Hence, the company should adopt strategy S_1 .

(iii) **Minimax Regret Criterion:** Opportunity loss table is shown below:

States of Nature	S_1	S_2	S_3
N_1	7,00,000 - 7,00,000 = 0	7,00,000 - 5,00,000 = 2,00,000	7,00,000 - 3,00,000 = 4,00,000
N_2	4,50,000 - 3,00,000 = 1,50,000	4,50,000 - 4,50,000 = 0	4,50,000 - 3,00,000 = 1,50,000
N_3	3,00,000 - 1,50,000 = 1,50,000	3,00,000 - 0 = 3,00,000	3,00,000 - 3,00,000 = 0
Column maximum	1,50,000	3,00,000	4,00,000
	↑ Minimax regret		

Hence, the company should adopt minimum opportunity loss strategy, S_1 .

(iv) **Laplace Criterion:** Since we do not know the probabilities of states of nature, assume that they are equal. For this example, we would assume that each state of nature has a probability $1/3$ of occurrence. Thus,

Strategy	Expected Return (Rs.)
S_1	$(7,00,000 + 3,00,000 + 1,50,000)/3 = 3,83,333.33$
S_2	$(5,00,000 + 4,50,000 + 0)/3 = 3,16,666.66$
S_3	$(3,00,000 + 3,00,000 + 3,00,000)/3 = 3,00,000$

Since the largest expected return is from strategy S_1 , the executive must select strategy S_1 .

Example 15.3 A manufacturer makes a product, of which the principal ingredient is a chemical X. At the moment, the manufacturer spends Rs. 1,000 per year on supply of X, but there is a possibility that the price may increase to four times its present figure because of a worldwide shortage of the chemical. There is another chemical Y, which the manufacturer could use in conjunction with a third chemical, Z in order to give the same effect as chemical X. Chemicals Y and Z would together cost the manufacturer Rs. 3,000 per year, but their prices are unlikely to rise. What action should the manufacturer take? Apply the maximin and minimax criteria for decision-making and give two sets of solutions. If the coefficient of optimism is 0.4, find the course of action that minimizes the cost.

Solution The data of the problem is summarized in the following table (negative figures in the table represents cost).

States of Nature	Courses of Action	
	S_1 (use Y and Z)	S_2 (use X)
N_1 (Price of X increases)	-3,000	-4,000
N_2 (Price of X does not increase)	-3,000	-1,000

Maximin Criterion:

States of Nature	Courses of Action	
	S_1	S_2
N_1	-3,000	-4,000
N_2	-3,000	-1,000
Column minimum	-3,000	-4,000

↑ Maximin

Minimum of column minima = -3,000. Hence, the manufacturer should adopt action S_1 .

Minimax (or opportunity loss) Criterion:

States of Nature	Courses of Action	
	S_1	S_2
N_1	$-3,000 - (-3,000) = 0$	$-3,000 - (-4,000) = 1,000$
N_2	$-1,000 - (-3,000) = 2,000$	$-1,000 - (-1,000) = 0$
Maximum opportunity	2,000	1,000 ← Minimax

Hence, manufacturer should adopt minimum opportunity loss course of action S_2 .

Hurwicz Criterion: Given the coefficient of optimism equal to 0.4, the coefficient of pessimism will be $1 - 0.4 = 0.6$. Then according to Hurwicz, select course of action that optimizes (maximum for profit and minimum for cost) the payoff value

$$H = \alpha (\text{Best payoff}) + (1 - \alpha) (\text{Worst payoff})$$

$$= \alpha (\text{Maximum in column}) + (1 - \alpha) (\text{Minimum in column})$$

Course of Action	Best Payoff	Worst Payoff	H
S_1	-3,000	-3,000	-3,000
S_2	-1,000	-4,000	-2,800

Example 15.10 A grocery with a bakery department is faced with the problem of how many cakes to buy order to meet the day's demand. The grocer prefers not to sell day-old goods in competition with fresh products; leftover cakes are, therefore, a complete loss. On the other hand, if a customer desires a cake and one of them has been sold, the disappointed customer will buy elsewhere and the sales will be lost. The grocer has therefore, collected information on the past sales on a selected 100 day period as shown in the table below

Sales per Day	No. of Days	Probability
25	10	0.10
26	30	0.30
27	50	0.50
28	10	0.10

A cake costs Rs. 80 and sells for Rs. 100. Construct the pay-off table and the opportunity loss table. What is the optimum number of cakes that should be bought each day?

Solution N_i ($i = 1, 2, 3, 4$) be all possible courses of action of stocking cakes and S_j ($j = 1, 2, 3, 4$) be the states of nature (daily likely demand).

$$\begin{aligned} \text{Conditional payoff} &= \text{MP} \times \text{Cakes sold} - \text{MP} \times \text{Cakes not sold} \\ &= \text{Rs. } (100 - 80) \times \text{Cakes sold} - \text{Rs. } 80 \times \text{Cakes not sold.} \\ &= \begin{cases} 20D & , \text{ if } D \geq S \\ 20D - 80(S - D) = 100D - 80S & , \text{ if } D < S \end{cases} \end{aligned}$$

where D denotes the number of units demanded and S is the number of units stocked.

The conditional pay-off values can be obtained as shown below:

Conditional Profit Values (Pay-offs)

State of Nature (Demand)	Probability	Conditional Pay-off (Rs.) Course of Action (Stock)			
		$A_1 : 25$	$A_2 : 26$	$A_3 : 27$	$A_4 : 28$
$E_1 : 25$	0.10	500	420	340	260
$E_2 : 26$	0.30	500	520	440	360
$E_3 : 27$	0.50	500	520	540	460
$E_4 : 28$	0.10	500	520	540	560
Expected Monetary Value (EMV)		500	510	490	420

The maximum EMV is seen in course of action A_2 . Thus, according to the EMV decision criterion, the store should stock 26 cakes. Calculations for EOL values are shown in the table below:

Opportunity Loss Values

State of Nature (Demand)	Probability	Conditional Opportunity Loss (Rs.) Course of Action (Stock)			
		$A_1 : 25$	$A_2 : 26$	$A_3 : 27$	$A_4 : 28$
$E_1 : 25$	0.10	0	80	160	240
$E_2 : 26$	0.30	20	0	80	160
$E_3 : 27$	0.50	40	20	0	80
$E_4 : 28$	0.10	60	40	20	0
Expected Opportunity Loss (EOL)		32	22	42	112

The minimum EOL is seen in course of action A_2 . Thus, according to the EOL decision criterion, the store should stock 26 cakes.